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Multiple Equilibria and Minimum Wages in Labor Markets with Informational Frictions and Heterogeneous Production Technologies

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Abstract

It is often argued that a mandatory minimum wage is binding only if the wage density displays a spike at it. In this paper we analyze a model with wage setting, search frictions, and heterogeneous production technologies, in which imposition of a minimum wage affects wages even though, after imposition, the lowest wage in the market exceeds the minimum wage, and subsequent abolition of the minimum wage does not affect wages. The model has multiple equilibria as a result of the fact that the reservation wage of the unemployed and the lowest production technology in use affect each other. Under certain conditions, imposition of a minimum wage improves social welfare.

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1 Introduction

The effect of a minimum wage on unemployment has been subject of a large number of empirical studies [1]. Recently, Card and Krueger (1995) have cast doubt on the perceived wisdom that this effect is positive. Although their results are somewhat controversial, they have been influential as well (see for example Kennan, 1995). To provide a theoretical explanation of a zero (or negative) effect, Card and Krueger (1995) hint at monopsony models of the labor market. Traditionally, the presence of monopsony power has been associated with geographically isolated markets, and as such the relevance of monopsony models has been put into question. However, it is by now well-known that the presence of informational frictions (or search frictions) on the labor market gives employers monopsony power as well.[2] Basically, if firms pay wages that are strictly smaller than the productivity level of the workers then they can still maintain a positive workforce and earn a profit, because it takes time for the workers to find a better paying job.

The imposition of a mandatory minimum wage reduces the degree to which employers can exploit their monopsony power. As long as the minimum wage does not exceed the productivity level, it merely redistributes part of the rents of the match from the firm to the worker, and as such it decreases the difference between marginal labor productivity and the wage. In a basic equilibrium search model framework, this shifts the whole wage distribution upward, but unemployment is not affected (see Van den Berg and Ridder, 1998). In more general frameworks, unemployment decreases if the upward shift of the wage distribution induces unemployed workers to accept jobs more frequently (see Burdett and Mortensen, 1998, and Bontemps, Robin and Van den Berg, 1999) or increase their search intensity. In the first case, the imposition of a minimum wage is unambiguously welfare-increasing.

In this context, a minimum wage can have the additional beneficial effect of driving out less productive firms. Consider a frictional labor market in which different firms have different production technologies. In equilibrium, the firms with a smaller marginal revenue product will offer lower wages than the firms with a higher marginal revenue product. Nevertheless, the former type of firms have a positive steady-state labor force and profits level. On the one hand, their employees will accept any job offer from a high-productivity firm. However, on the

[1] See Brown, Gilroy and Kohen (1982) for a survey of research up to the early eighties and Card and Krueger (1995) for a recent survey.

[2] See Boal and Ransom (1997) for a recent survey on monopsony.

other hand, there will always be unemployed workers who prefer a temporary job at a low-productivity firm over a prolonged spell of unemployment (see Bontemps, Robin and Van den Berg, 2000). Thus, these low-productivity firms can coexist with other firms. However, imposition of a sufficiently high minimum wage will make these low-productivity firms go bankrupt. As a result, after imposition, workers will only be employed at high-productivity firms, and total social welfare may increase (see Eckstein and Wolpin, 1990).

All of the minimum wage effects mentioned above are comparative-statics results derived for equilibrium search models of the labor market (see below for a more detailed discussion of the literature). In this paper we examine a minimum wage effect that has not been detected before. This effect follows from the existence of *multiple equilibria* on the labor market, in the context of informational frictions and dispersion of firms' production technologies. To understand the existence of multiple equilibria intuitively, note first of all that the wage offer by a firm must be in between the reservation wage of the unemployed and the productivity level of the firm. (If the offer falls short of the reservation wage then no workers can be attracted.) The idea behind the multiplicity of equilibrium is then as follows. Basically, a labor market has *either (1)* high-productivity as well as low-productivity firms, with unemployed workers using a low reservation wage, *or (2)* high-productivity firms only, with unemployed workers using a high reservation wage. In the second equilibrium, the high reservation wage acts as a binding lower bound on the set of production technologies that enable a positive profit per worker. It rules out production at a low productivity level. Now, in general (i.e., in both equilibria), part of the rent of production is distributed to the workers in the form of the wage. The resulting wage distribution for the second equilibrium dominates the wage distribution in the first equilibrium. This in turn justifies the higher reservation wage in the second equilibrium. Thus, the reservation wage of the unemployed affects the set of profitable production technologies, while the set of production technologies in use in turn affects the reservation wage value, and this system of highly nonlinear equations has multiple solutions.[3]

[3] Acemoglu (1997) derives a result on multiplicity of steady-state equilibria in a matching model. In that model (contrary to the equilibrium search model in the present study), a firm is effectively equivalent to a single job, firms endogenously choose their production technology prior to production, wages follow from decentralized bargaining, workers cannot search on the job, and labor market tightness depends on the unemployment rate. Multiple equilibria can occur because the outside option of workers who find a bad job depends on the rate at which they can find a good job; if the latter is high then this may drive the wage of the bad job up to an unprofitable level. The precise conditions for multiplicity are very different from ours. For

Suppose that the labor market is in the first equilibrium, with on average lower wages and less efficient production than in the second equilibrium. Now consider the imposition of a minimum wage exceeding **the** productivity level of the low-productivity firms. The latter firms will go bankrupt, and the resulting equilibrium is the second equilibrium, with on average higher wages and more efficient production. (In a basic model, there is no effect on unemployment at all.) If the minimum wage is subsequently abolished, the labor market may not deviate from the (second) equilibrium. In that case, a temporary imposition of (or increase in) the minimum wage is sufficient to force the labor market from the unfavorable to the favorable equilibrium.

The empirical research on minimum wage effects generally assumes that for a minimum wage to be binding, it is necessary that the wage density displays a spike at the minimum wage. This spike is thought to capture jobs with productivity at or above the minimum wage as well as below the minimum wage. The latter jobs may exist temporarily because of job protection legislation or because of the fact that factor substitution and investment take time. If a spike is absent then, by analogy to the competitive model, it is argued that workers and wages are only marginally affected by a change in the minimum wage. The analysis in the present paper has radically different implications. First, if the imposition of a minimum wage changes the equilibrium outcome, then the wage density in the new equilibrium does not necessarily have a spike at the lowest wage. Indeed, the lowest wage can be strictly larger than the mandatory minimum wage. Our results are thus consistent with data on repeated cross-sections showing that the minimum wage level affects the shape of the wage density even though the latter does not have a spike at the minimum wage level (see e.g. Östros, 1994, who examines Swedish data). Secondly, if there are multiple equilibria then the abolishment of the minimum wage may not affect the equilibrium. This could lead to the erroneous conclusion that the minimum wage was therefore irrelevant. Note that with multiple equilibria, the notion of "binding" becomes somewhat unclear.

During the past decade, the theoretical and empirical analysis of equilibrium search models has become widespread (see surveys by Van den Berg, 1999, and Mortensen and Pissarides, 1999). Most of this literature builds on Burdett and Mortensen (1998), who develop a model with homogeneous workers and firms in which workers are allowed to search on the job, and on Mortensen (1990), who extends this model by introducing heterogeneity. In the homogeneous model, the possibility of on-the-job search is a sufficient condition for wage dispersion

example, in our main model, equilibrium is always unique if on-the-job search is impossible.

in equilibrium. In that case, job-to-job transitions are important for maintaining the workforce of a firm. However, it is generally acknowledged that, to explain the shape of the wage distribution, it is necessary to allow for heterogeneity in production levels as well (see Van den Berg and Ridder, 1998, and Bontemps, Robin and Van den Berg, 2000). The resulting models satisfy a large number of stylized facts of the labor market, particularly concerning the relations between job durations, wages, and the sizes of firms (see e.g. Ridder and Van den Berg, 1997).

In this paper, we adopt the benchmark model developed by Mortensen (1990), in which workers search on the job and production technologies are dispersed across firms.[4] Indeed, for expositional reasons, we will mostly assume that there are two possible productivity levels. This model is sufficiently rich for our purposes. If both types of firms are active then unemployed and employed workers receive job offers from both types, and workers at low-productivity firms move to high-productivity firms whenever they get the opportunity.

The model by Mortensen (1990) with heterogeneous firms and homogeneous workers has been popular in the empirical analysis of equilibrium search. For example, Bowlus, Kiefer and Neumann (1995, 2000), Bowlus (1997), and Bunzel et al. (1999) estimate this model allowing for a finite number of different firm types. They argue that in general a rather small number of firm types gives a reasonable fit to the main quantiles of the wage distribution. It should be stressed that none of this literature has addressed the issue of multiplicity of equilibrium, or for that sake the possibility that equilibrium may switch in response to policy changes. Mortensen (1990) derives many properties of the equilibrium solutions. Bontemps, Robin and Van den Berg (2000) analyze a model with a *continuous* distribution of different production technologies. This model is able to give a perfect fit to wage data, but due to its complexity it is less amenable to a formal analysis of conditions for multiplicity of equilibrium.

The outline of the paper is as follows. In Section 2 we present the model. In Section 3 we derive the equilibria under the assumption that the job offer arrival rates for the workers do not depend on the measure of active firms in the economy. This is a strong assumption. Arrival rates depend on the matching technology in the labor market, and if the measure of firms decreases then the matching rate may decrease as well. In Section 4 we therefore extend the model to deal with this. This extension has not been examined in the above-mentioned literature on equilibrium search. It turns out that the results are robust with respect to this.

[4]Mortensen (1990) also allows for heterogeneity of workers' opportunity costs of employment (see below).

In Sections 3 and 4 we also discuss the empirical relevance of multiple equilibria, by examining some cases from the empirical literature. In particular, we show that the estimates in Bunzel et al. (1999) for specific labor markets are such that these markets display multiple equilibria. In Subsection 3.4 we show that the results derived for the model with two productivity levels can be extended to models with more general productivity distributions. Section 5 deals with the effects of changes in the minimum wage. In Section 6 we examine an alternative model in which unemployed workers are heterogeneous with respect to the level of their unemployment benefits. This model generates multiple equilibria as well, for essentially the same reason as in the basic model. Section 7 concludes.

2 The model framework

Because the model framework is discussed in Mortensen (1990) as well as in subsequent empirical studies, the present exposition can be brief. The model considers a labor market consisting of a continuum of workers and firms. The measure of workers is denoted by m, and the measure of unemployed workers by u. In Mortensen (1990), the measure of firms is normalized to one. Here, we must be more explicit on this. In a given steady-state equilibrium, there can be active (profitable) firms as well as non-active latent firms that may be active in another equilibrium. We assume that the total measure of firms (active or potentially active) is fixed, for example because it is determined by capital endowments, and we denote this measure by n. Later on we will see that if all unemployed workers accept any nonnegative wage offer then the measure of *active* firms is also equal to n.

The supply side of the model is equivalent to a standard partial job search model with on-the-job search (see Mortensen, 1986). Workers obtain wage offers, which are random drawings from the wage offer distribution $F(w)$, at an exogenous rate λ_0 when unemployed and λ_1 when employed. Whenever an offer arrives, the decision has to be made whether to accept it or to reject it and search further for a better offer. Firms post wage offers and they do not bargain over the wage. Layoffs accrue at the constant exogenous rate δ. The opportunity cost of employment is denoted by b and is assumed to be constant across individuals and to be inclusive of unemployment benefits and search costs. We take $0 < \lambda_0, \lambda_1, \delta <$ oo and 6 ≥ 0.[5] The optimal acceptance strategy for the

[5]For expositional reasons we restrict attention to the limiting case in which the workers' discount rate is infinitesimally small. The results are robust with respect to this.

unemployed is then characterized by a reservation wage ϕ satisfying

$$\phi = b + (\lambda_0 - \lambda_1) \int_{\phi}^{\infty} \frac{1 - F(w)}{\delta + \lambda_1(1 - F(w))} \, dw \tag{1}$$

It is not difficult to show that this equation does give a unique solution for ϕ given the other variables and functions![6]

Employed workers accept any wage offer that exceeds their current wage. In sum, workers climb the job ladder to obtain higher wages, but this effort may be frustrated by a temporary spell of frictional unemployment.

Now consider the flows of workers. First, note that active firms do not offer a wage below ϕ, so that all wage offers will be acceptable for the unemployed. Consequently, the flow from unemployment to employment is $\lambda_0 u$. The flow from employment to unemployment is $\delta(m - u)$. In a steady state these flows are equal and the resulting rate u/m of unemployed workers equals S/(6 + λ_0).

Let the distribution of wages paid to a cross-section of employees have distribution function G. These wages are on average higher than the wages offered, because of the flow of employees to better paying jobs. The stock of employees with a wage less or equal to w has measure G(w) $(m - u)$. The flow into this stock consists of unemployed who accept a wage less than or equal to w, and this flow is equal to $\lambda_0 F(w) u$ The flow out of this stock consists of those who become unemployed, $\delta G($ w) $(m -$ u) and those who receive a job offer that exceeds w, $\lambda_1(1 - F(w))G(w)(m -$ **u).** In the steady state, the flows into and out of the stock are equal, so

$$G(w) = \frac{\delta F(w)}{\delta + \lambda_1(1 - F(w))} \tag{2}$$

where we have substituted for u.

From the two wage distributions we derive the steady-state supply of labor $l(w|\ F)$ to an employer setting a wage w, where we explicitly indicate its dependence on the wages offered by other firms. Somewhat loosely, one may say that this must equal the number of workers earning w in a steady state, divided by the number of firms paying w in the steady state. Note that it is assumed that a firm pays the same wage to all of its employees. Let n_a denote the measure of active firms (this is not a fundamental model determinant but rather an equilibrium outcome) .[7] As a result,

[6]The derivative of the left-hand side with respect to ϕ equals one. If $\lambda_0 \geq \lambda_1$ then the derivative of the r.h.s. w.r.t. ϕ is negative or zero. If $\lambda_0 < \lambda_1$ then the derivative of the r.h.s. w.r.t. ϕ is positive and uniformly smaller than one.

[7]As mentioned in the introduction, λ_0 and λ_1 may depend on n_a. We deal with this in Section 4.

$$l(w|F) = m \frac{\bar{}}{n_a} u \frac{\delta(\delta+\lambda_1)}{(\delta+\lambda_1(1-F(w)))^2} - \frac{m\delta\lambda_0(\delta+\lambda_1)}{n_a(\delta+\lambda_0)} \frac{1}{(\delta+\lambda_1(1-F(w)))^2} \tag{3}$$

See Bontemps, Robin and Van den Berg *(2000)* for a formal analysis. The equation above only holds if F does not have mass points, which is shown below. Further, in this equation, w has of course to exceed ϕ (we do not consider a mandatory minimum wage until Section 5). It is easily seen that l increases in w on the support of F.

Now consider a firm with a flow p of marginal revenue product generated by employing one worker. For convenience, we assume that p does not depend on the number of employees, i.e. we assume that the production function is linear in employment. We refer to this firm as a firm of type p and to p as *the* (labor) productivity of this firm. Each firm sets a wage w so as to maximize its steady-state profit flow $(p-w)l(w|F)$ given F and given the behavior of workers.

We distinguish between two types of firms in this labor market. Firms of type p_2 (p_1) have a production technology that gives them a low (high) labor productivity p_2 (p_1), with $p_1 > p_,$. It should be stressed that this productivity level is a firm characteristic and not a worker characteristic. One may think of a market in which individuals with a certain level of education are employable in two different occupations. A fraction q of the total measure of firms (active or potentially active) consists of type-p, firms, and the remaining fraction $1-q$ consists of type-p, firms. We take q (and therefore the measures both types of firms qn and $(1-q)n)$ to be fixed. For example, these may have been determined by capital endowments. Indeed, Acemoglu and Shimer (1997) show that productivity dispersion can be explained as an equilibrium outcome by letting ex ante homogeneous firms choose their capital before production starts. Alternatively, productivity dispersion may be the result of differences in product market power. We take $p_1 > b$ and $0 < q < 1$. If $p_1 < b$ then there would be no production, as all workers would be better off by being unemployed.

Mortensen (1990) derives a number of properties of any equilibrium. First of all, a wage offer w that attracts workers is profit maximizing only if no mass of other employers offer w. Consequently, the equilibrium wage offer distribution F has no mass point. This is a fundamental property of equilibrium search models with on-the-job search. It follows from the fact that a mass point at say w^* induces any firm paying w^* to offer a wage slightly higher than w^*, because then its steady-state labor force will be substantially larger at the cost of only a second order decrease in the profit per worker. This property implies that all workers face a non-degenerate wage offer distribution, and job-to-job transitions do occur. Moreover, any equilibrium is asymmetric in pure (firms') strategies (or symmetric in mixed (firms') strategies). That is, equivalent firms offer different wages, yet they receive the same profit flow. Note that the function $l(w|F)$ increases in w, so there is a positive relation between the size of the firm and the wage it offers. A large (small) wage implies that the exit rate of workers at the firm is relatively small (large), and that a relatively large (small) fraction of all workers currently employed in the economy is willing to work at the firm. Hence, in terms of total profits of a firm, there is a trade-off between the profit per worker and the steady-state number of workers at the firm.[8]

A second property is that profit-maximizing wages for type-p, employers are larger than profit-maximizing wages for type-p, employers, if both types are active, and that there is no gap between the corresponding parts of the support of F. Indeed, firms with a higher labor productivity offer higher wages, have a larger labor force and have larger profit flows than firms with lower labor productivity. A third property is that the lowest wage in the market $\underline{w}$ equals the reservation wage ϕ of the unemployed. If the lowest wage would exceed the reservation wage then the firm offering that lowest wage would increase its profit per worker by lowering the offer, without any loss in its steady state labor force. Note that a firm always offer a wage that is smaller than its productivity level, since they can always attain a positive profit by offering the reservation wage of the unemployed.

3 Equilibria

In this section we show that the model of Section 2 can have multiple equilibria. In one equilibrium, all active firms are type-p, firms, whereas in the other, both types are active. We are concerned with non-cooperative steady-state equilibria. Somewhat loosely, such equilibria can be thought to consist of a reservation wage ϕ and a wage offer distribution F such that (i) ϕ satisfies (1) given F, and (ii> F follows from the firms' maximizations of their own steady-state profit flows.

[8] In this respect, there is a strong similarity to "turnover costs" efficiency wage models; see e.g. Stiglitz (1985) and Weiss (1991). The model setup also shares features with Reinganum (1979), who studies equilibrium search in consumer product markets, and with Montgomery (1991), who studies a one-shot model of equilibrium search with productivity dispersion on the labor market. See Ridder and Van den Berg (1997), Acemoglu and Shimer (1997) and Montgomery (1991) for overviews of the empirical evidence supporting these types of models.

3.1 Only high-productivity firms

We start by assuming that only type-p, firm types are active in equilibrium. This must be verified, by checking whether in equilibrium $\phi \geq p_2$. **If** the latter does not hold then there is an incentive for type-p, firms to enter the market. We derive the equilibrium solution as follows: first derive the wage offer distribution for a given unknown $\phi < p_1$; then calculate the actual reservation wage by substituting the wage offer distribution into equation **(1).** Finally, check whether the resulting ϕ exceeds p_2.

If only type-p, firms are active then we have a model with homogeneous firms. The equilibrium in such a model has been solved many times in the literature (see e.g. Burdett and Mortensen, 1998). Note that here the measuren, of active firms equals qn. For a given $\phi < p_1$, the steady-state profit flow of a firm offering ϕ equals $(p_1 - \phi)l(\phi|F)$. Using equation (3), this in turn equals

$$(p_1 - \phi)\frac{m\delta\lambda_0}{q\,n\,(\delta+\lambda_0)(\delta+\lambda_1)}$$

In equilibrium, all other firms have the same profit flow. Thus, all profit-maximizing wages w satisfy the equality of $(p_1 - w)l(w|F)$ to the expression above, with $l(w|F)$ as in (3). This can be solved for F,

$$F(w) = \frac{\delta+\lambda_1}{\lambda_1}\left(1 - \sqrt{\frac{p_1 - w}{p_1 - \phi}}\right) \tag{4}$$

F has support $(\phi, \overline{w})$, with

$$\overline{w} = \left(\frac{\delta}{\delta+\lambda_1}\right)^2 \phi + \left(1 - \left(\frac{\delta}{\delta+\lambda_1}\right)^2\right) p_1 \tag{5}$$

Obviously, this lies between ϕ and p_1. Furthermore,

$$G(w) = \frac{\delta}{\lambda_1}\left(1 - \sqrt{\frac{p_1 - \phi}{p_1 - w}}\right) \tag{6}$$

and

$$l(w|F) \quad = \quad \frac{m\delta\lambda_0 \qquad\qquad p_1 - \phi}{q\,n\,(\delta+\lambda_0)(\delta+\lambda_1)\; p_1 - \qquad -} \tag{7}$$

By substituting **(4)** into **(l),** it follows that

$$\phi = \frac{(\delta + \lambda_1)^2 b + (\lambda_0 - \lambda_1)\lambda_1 p_1}{(\delta + \lambda_1)^2 + (\lambda_0 - \lambda_1)\lambda_1} \tag{8}$$

This satisfies $\phi \geq p_2$ if and only if

$$(\lambda_0 - \lambda_1)\lambda_1(p_1 - p_2) \geq (\delta + \lambda_1)^2(p_2 - b) \tag{9}$$

In sum,

Proposition 1 *There is an equilibrium in which only type-p, firms are active if and only* ***if*** $(\lambda_0 - \lambda_1)\lambda_1(p_1 - p_2) \geq (\delta + \lambda_1)^2(p_2 - b)$.

A number of comments are in order. First of all, note that the condition (9) does not depend on q or on n_a or n or m. Secondly, the condition can be made more transparent by examining special cases. For example, the equilibrium above does not exist if both $p_2 \geq b$ and $\lambda_1 > \lambda_0$. This makes sense: if search in employment is more efficient than search in unemployment then an unemployed worker accepts a job with a wage equal to b, so then type-p, firms can make a positive profit. Other special cases are discussed in Subsection 3.3. There we also discuss the empirical plausibility of the condition.

3.2 High-productivity firms as well as low-productivity firms

Now suppose that firms of both types are active in equilibrium. This must again be verified, by checking whether in equilibrium $\phi < p_2$. If the latter does not hold then type-p_2 firms will disappear from the market.

The expressions for the equilibrium in such a model follow from the equilibrium properties listed in Section 2. Note that here the measure n_a of active firms equals n. The support of F consists of two adjacent parts, say $(\phi, \widehat{w})$ and $(\widehat{w}, \overline{w})$. For a given $\phi < p_2$, the steady-state profit flow of a type-p, firm offering ϕ equals $(p_2 - \phi)l(\phi|F)$. Using equation (3), this in turn equals

$$(p_2 - \phi)\frac{m\delta\lambda_0}{n\,(\delta + \lambda_0)(\delta + \lambda_1)}$$

In equilibrium, all other type-p, firms have the same profit flow. Thus, all profit-maximizing wages w of type-p, firms satisfy the equality of $(p_2 - w)l(w|\ F)$ to the expression above, with $l(w|F)$ as in (3). This can be solved for F(w) on $w \in (\phi, \widehat{w})$,

$$F(w) = \frac{\delta + \lambda_1}{\lambda_1}\left(1 - \sqrt{\frac{p_2 - w}{p_2 - \phi}}\right) \tag{10}$$

where $\widehat{w}$ follows from the restriction that $F(\widehat{w}) = 1 - q$. As a result,

$$\widehat{w} = \left(\frac{\delta + \lambda_1 q}{\delta + \lambda_1}\right)^2 \phi + \left(1 - \left(\frac{\delta + \lambda_1 q}{\delta + \lambda_1}\right)^2\right) p_2 \tag{11}$$

This lies between ϕ and p_2. Note that if $q = 0$ and $p_2 = p_1$ then the expression above equals $\overline{w}$ in (5), as it should.

Now let us turn to type-pi firms. For a given ϕ, and for given behavior of type-p, firms, the steady-state profit flow of a type-pi firm offering $\widehat{w}$ equals $(p_1 - \widehat{w})l(\widehat{w}|F)$. Using equation (3), this equals

$$(p_1 - \widehat{w})\frac{m\delta\lambda_0(\delta + \lambda_1)}{n\,(\delta + \lambda_0)(\delta + \lambda_1 q)^2}$$

In equilibrium, all other type-p, firms have the same profit flow. Thus, all profit-maximizing wages w of type-p, firms satisfy the equality of $(p_1 - w)l(w|F)$ to the expression above, with $l(w|F)$ as in (3). This can be solved for $F(w)$ on $w \in (\widehat{w}, \overline{w})$,

$$F(w) = \frac{\delta + \lambda_1}{\lambda_1}\left(1 - \frac{\delta + \lambda_1 q}{\delta + \lambda_1}\sqrt{\frac{p_1 - w}{p_1 - \phi}}\right) \tag{12}$$

where $\overline{w}$ follows from $F(\overline{w}) = 1$, with F as above,

$$\overline{w} = \left(\frac{\delta}{\delta + \lambda_1 q}\right)^2 \widehat{w} + \left(1 - \left(\frac{\delta}{\delta + \lambda_1 q}\right)^2\right) p_1$$

which can be rewritten using the equation for $\widehat{w}$,

$$\overline{w} = p_1 - (p_1 - p_2)\left(\frac{\delta}{\delta + \lambda_1 q}\right)^2 - (p_2 - \phi)\left(\frac{\delta}{\delta + \lambda_1}\right)^2 \tag{13}$$

Equations (10)–(13) constitute F given ϕ. It is important to realize that, here as well as in the previous subsection, the shape of F reflects the bargaining power of workers vis-a-vis employers. A lower degree of search frictions for employed job seekers (i.e. a high X,/S) provides an incentive for firms to pay higher wages.

If search frictions are lower, then a firm paying a higher wage would be able to increase its labor force substantially (see Ridder and Van den Berg, 1997).

By substituting (10)–(13) into (1), and after tedious calculations, it follows that

$$\phi = \frac{(\delta + \lambda_1)^2 b + (\lambda_0 - \lambda_1)\lambda_1 (cp_1 + (1 - c)p_2)}{(\delta + \lambda_1)^2 + (\lambda_0 - \lambda_1)\lambda_1} \tag{14}$$

with c defined as

$$c = \left(\frac{(\delta + \lambda_1)q}{\delta + \lambda_1 q}\right)^2$$

Note that $0 < c < 1$ and that the denominator of ϕ is positive. The expression for ϕ is similar to the expression (8) for the homogeneous model. The only difference is that the productivity level in the homogeneous model is replaced by a weighted average of the productivities in the present model. The weights c and $1 - c$ reflect the relative importance of p_1 and p_2 for the unemployed individual.[9] In general, c does not equal q. This is because firms take firm heterogeneity into account when they set wages, and because workers can move from low-productivity firms to high-productivity firms. If $q = 0$ ($q = 1$) then $c = 0$ ($c = 1$) and ϕ equals the reservation wage in the homogeneous model with $p = p_2$ ($p = p_1$). The weight c increases in q and λ_1/δ. This makes sense, as a high value of q means that high-productivity jobs are abundant, while a high value of λ_1/δ means that it is relatively easy to move quickly to a job with a high productivity.

It now remains to check whether $\phi < p_2$. This can be shown to hold if and only if

$$(\lambda_0 - \lambda_1)\lambda_1 c(p_1 - p_2) < (\delta + \lambda_1)^2 (p_2 - b) \tag{15}$$

In sum,

Proposition 2 *There is an equilibrium in which both types of firms are active if and only if* $(\lambda_0 - \lambda_1)\lambda_1 c(p_1 - p_2) < (\delta + \lambda_1)^2 (p_2 - b)$.

[9] This can be seen from the expression for the mean wage $E_G(w)$ earned by employed workers for a given lowest wage $\underline{w} < p_2$,

$$E_G(w) = \frac{\delta}{\delta + \lambda_1}\underline{w} + \frac{\lambda_1}{\delta + \lambda_1}(cp_1 + (1 - c)p_2)$$

Since workers do not discount the future, $E_G(w)$ is the expected steady-state income flow in employment for a currently unemployed worker.

By combining Propositions 1 and 2 we obtain

Proposition ***3 There exist exactly two equilibria if and only if***

$$\frac{(\lambda_0 - \lambda_1)\lambda_1}{(\delta + \lambda_1)^2} \geq \frac{p_2 - b}{p_1 - p_2} > \frac{(\lambda_0 - \lambda_1)\lambda_1}{(\delta + \lambda_1)^2} c \tag{16}$$

Note that λ_0, λ_1 and δ only affect these inequalities by way of the ratios λ_0/δ and λ_1/δ. Also note that the middle term only depends on monetary flow variables whereas the left-hand and right-hand sides only depend on time rates (though the right-hand side also depends on the fraction q of high-productivity firms). This implies for example that if $\lambda_0 > \lambda_1$ then there are always values of b, p_1, p_2 such that there are multiple equilibria.

Figure 1 displays the admissible equilibrium combinations of ϕ and the lowest productivity among active firms in the market. If p_1 is the lowest p among active firms then all values of ϕ in between p_2 and p_1 are feasible, whereas if p_2 is the lowest p among profitable firms then all values of ϕ below p_2 are feasible. In general, it cannot be ruled out that both line segments contain an equilibrium point, or that neither contains such a point.

If (16) is satisfied then the unique values of the structural determinants support two different equilibria, and the model does not predict which one will be realized and which one will not. As explained in the introduction, this is an implication of the interaction between the reservation wage of the unemployed and the distribution of technologies that admit profitable production. The reservation wage ϕ affects the distribution of p among active firms by way of the restriction that these p should exceed ϕ. Thus, ϕ affects the ***lower bound of the support*** of the distribution of p among active firms. Conversely, the distribution of p among active firms affects the reservation wage, because part of the rent of production is distributed to the workers in the form of the wage. This effect works by way of the ***expected*** wage in employment. Due to the nonlinearity of the relation between w and p, the reservation wage ϕ does not depend on the expectation of p, but rather on the ***expectation of a monotone transformation*** of p among ***active firms.*** If the distribution of p shifts to the right then the wage (offer) distribution shifts to the right, and if workers search more easily while unemployed than while employed (i.e., $\lambda_0 > \lambda_1$) then this implies a higher reservation wage: it makes sense to be more selective while unemployed.

In the next subsection we go into more detail concerning the cases in which multiple equilibria may occur, and their respective empirical relevance.

3.3 Discussion

First of all, it is useful to distinguish between whether λ_0 is smaller than, equal to, or larger than λ_1. If they are equal, then search while unemployed is as effective as search while employed, and consequently the reservation wage of the unemployed is equal to the instantaneous utility flow while being unemployed, which is the value of leisure b. This means that there is no feedback from the productivity (or wage) distribution to the reservation wage, and the equilibrium is unique. From the propositions above, only type-p, firms are active iff $p_2 \leq b$, while both types are active iff $p_2 > b$, which is intuitively obvious. Thus,

If $\lambda_0 = \lambda_1$ then there exists a unique equilibrium.
If $b \geq p_2$ then only type-p, firms are active. If $b < p_2$ then both types are active.

Now consider the case $\lambda_0 > \lambda_1$. Then workers search more easily while unemployed than while employed, so it makes sense to be more selective while unemployed than if λ_0 were equal to λ_1. Consequently, the reservation wage exceeds b. Moreover, as argued in the previous subsection, higher wages are associated with a higher reservation wage. By elaborating on (16) we obtain

If $\lambda_0 > \lambda_1$ then there always exists an equilibrium.
If $b \geq p_2$ then the equilibrium is unique (only type-p, firms are active). If $b < p_2$ then we have either one of the following three situations: equilibrium is unique and only type-p, firms are active, equilibrium is unique and both types are active, or both equilibria are possible. All three cases are attainable for suitable parameter values.

The empirical evidence in the literature suggests that in general $\lambda_0 > \lambda_1$. For example, Ridder and Van den Berg (1998) estimate these parameters for a number of OECD countries, and they find that, typically, λ_0 is about two to five times as large as λ_1 (see also Bontemps, Robin and Van den Berg, 2000, for a discussion of empirical results in the literature). Indeed, for plausible parameter values, there are generally multiple equilibria. For example, a frictional unemployment rate of 6.25% corresponds to $\lambda_0/\delta = 15$. If in addition $\lambda_0 = 3\lambda_1$ then the left-hand side of (16) equals about 1.4. It is not unreasonable to expect the middle term of (16) to be smaller than that. For example, if b be close to the unemployment benefits level, and the worker can choose between two occupations, one which has a productivity level which is 100 monetary units per month larger than b and the other 200, then the middle term equals one. As a result, we have multiple equilibria, except for values of c close to one. In terms of q, *we* have multiple equilibria for all values of q smaller than about 0.5. So, in this example, if the productivity distribution is skewed to the right (there are more firms with low-productivity jobs than with high-productivity jobs) then there are multiple equilibria.

Example 1. To be more specific, take $\lambda_0 = 0.15$, $\lambda_1 = 0.05$, $\delta = 0.01$, $b = 1000, p_2 = 1100, p_1 = 1200, q = 0.25$ (to fix thoughts, these may be in Dutch guilders and months). Then in one equilibrium $\phi = 1084$, $\overline{w} = 1180$, and both types of firms are active, whereas in the other $\phi = 1116$, $\overline{w} = 1198$, and only type-p, firms are active. In both cases $u/m = 0.0625$.

Example 2. Bunzel et al. (1999) estimate equilibrium search models for a number of different Danish labor markets, assuming that $\boldsymbol{p}$ has a discrete distribution. The different markets are defined by the properties of the workers in it. Each market is assumed to have its own firms. One of the labor markets contains the workers with a bachelor degree who are aged between 22 and 30. This includes about 2.5% of the Danish population aged between 16 and 75. For this market, the estimation procedure produces two different mass points for $\boldsymbol{p}$ (there is no evidence for a third mass point). The structural parameters are estimated to be $\lambda_0 = 0.034$, $\lambda_1 = 0.009$, $\delta = 0.013$, $b = -42.8, p_2 = 217, p_1 = 1239$, $q = 0.129$ (monetary flows are in 1981 Danish Kroner per hours; time rates are in weeks). Bunzel et al. (1999) report the corresponding estimates of ϕ and $\overline{w}$ to be 54 and 325, respectively. The structural parameters also support another equilibrium, with $\phi = 364$ and $\overline{w} = 933$. In both cases, $u/m = 0.28$. Note that the data identify the first-mentioned equilibrium.

In their empirical analysis, Bunzel et al. (1999) ignore mandatory minimum wages. There are actually two types of minimum wages in Denmark (see Jensen, Rosholm and Smith, 1994).[10] The values of the highest of these for salaried workers are generally close to the observed lowest wages in the labor markets in Bunzel et al. (1999). If the lowest wage is equal to a minimum wage then the reservation wage ϕ is unidentified; we only know that it is smaller than or equal to the observed lowest wage. In that case, the reported estimates of ϕ and b could be upward biased, and there might actually be less scope for multiple equilibria.

[10] Each broadly-defined sector (like the metal sector) has a minimum wage that is determined in central wage negotiations between trade unions and employer organizations. In addition, each firm has minimum wages that are determined in negotiations between the firms and local branches of the relevant unions.

Recall however that the reported estimate of b is already very low (-42.8, whereas observed wages are dispersed in the interval $[54, 325]$ and the observed mean wage offer equals **114).** From Pedersen and Smith (1998) it follows that the minimum income support level (social assistance/welfare, including discretionary benefits and housing benefits) in Denmark in the early 1980s generally exceeds **15.** It therefore seems unlikely that ϕ and b are substantially smaller than estimated. In fact, they could be under-estimated in case of spurious dispersion of observed wages due to measurement errors.

For most other markets, Bunzel et al. (1999) find evidence of more than two mass points. We turn to this in Subsection 3.4.

It is instructive to consider the case $\lambda_0 < \lambda_1$ as well, even though this seems to be empirically less relevant, and even though, as we will see shortly, there are no multiple equilibria. In this case, workers search less easily while unemployed than while employed. Consequently, the reservation wage is smaller than b. The higher the wages, the more important it is to leave unemployment as quickly as possible, so the lower the reservation wage. By elaborating on **(16)** we obtain

If $\lambda_0 < \lambda_1$ then equilibrium is unique or nonexistent.
If $b < p_2$ then the equilibrium is unique (both types of firms are active). If $b \geq p_2$ then we have either one of the following three situations: equilibrium is unique and only type-p_1 firms are active, equilibrium is unique and both types are active, or equilibrium is nonexistent. All three cases are attainable for suitable parameter values.

From the intuition behind the multiplicity of equilibria in the case $\lambda_0 > \lambda_1$, it is obvious that multiplicity is not to be expected in this reverse case. Necessary conditions for nonexistence are that $\lambda_0 < \lambda_1$ and $b \geq p_2$. As will be discussed below, the latter is unlikely to occur in an economy in which a minimum wage is imposed that exceeds the unemployment benefits level.

It may be interesting to examine a few limiting cases. For p_2 sufficiently close to p_1, the equilibrium is unique (with both types of firms active). In the limit, of course, the equilibrium solution is equal to the unique solution for the homogeneous model (which is the equilibrium if only type-p, firms are active).

Now consider the limiting case $\lambda_1 = 0$, *i.e.* the employed do not receive alternative job offers. It is then optimal for wage-setting firms to offer a wage equal to the reservation wage of the unemployed. Offering a higher wage would not increase their workforce. The resulting equilibrium is then the same as in the model in which one firm is monopolist in the labor market: all firms offer a wage equal to the value of leisure b (the "monopolistic solution"; see Diamond, **1971).** Thus, we have the equilibrium in which only type-p_1 firms are active if $b \geq p_2$, and the other equilibrium if $b < p_2$. In any case, equilibrium exists and is unique.

Finally, consider what happens if search frictions vanish. If λ_0 approaches ∞, i.e. if the unemployed find jobs instantaneously, then they can afford to be extremely selective with respect to wage offers. As a result, ϕ approaches p_1, so only type-p_1 firms are active and the equilibrium is unique. In the limit, the equilibrium wage offer and wage distributions are degenerate in p_1. If λ_1 approaches infinity, i.e. if the employed find jobs instantaneously, then workers instantaneously move to the top of the wage ladder, and the wage distribution G approaches the degenerate distribution at p_1. In this case ϕ does not approach p_1, and neither does F. However, this is irrelevant, because an unemployed worker, upon leaving unemployment, immediately moves to a wage p_1. As a result, firm profits tend to zero. It follows from (16) that this equilibrium, which can be labeled the "competitive solution", is unique, and is such that both types of firms are active, although the type-p_2 firms lose their employees instantaneously.

Let us return to the situation in which two equilibria are possible. The equilibrium with only type-p_1 firms is to be preferred, since employed workers are on average more productive in this equilibrium, whereas unemployment is the same in both equilibria.[11] An unemployed worker contributes b to social welfare, whereas an employed worker together with his employer contributes the rent of the match, which equals the productivity level. Alternatively, they contribute the productivity level minus b (the first case can occur if the non-monetary value of leisure is zero and unemployment benefits are financed externally). For expositional convenience we adopt the first case; however, the results are robust with respect to this. In the equilibrium with only type-p_1 firms, the social welfare flow S^* then equals

$$S^* = \frac{\delta m}{\delta + \lambda_0} b + \frac{\lambda_0 m}{\delta + \lambda_0} p_1$$

In the other equilibrium, the social welfare flow S^{**} equals

$$S^{**} = \frac{\delta m}{\delta + \lambda_0} b + \frac{\lambda_0 m}{\delta + \lambda_0} \left[G(\hat{w})p_2 + (1 - G(\hat{w}))p_1\right]$$

where G denotes the cross-sectional wage distribution among workers. Clearly, $S^{**} < S^*$.

[11] The assumption that production is linear in the number of employees is important here; decreasing returns to scale would mitigate the effect.

Using equation (2), $G(\hat{w})$ can be expressed in terms of $F(\hat{w})$, which equals $1 - q$. As a result, S^{**} equals

$$S^{**} = \frac{\delta m}{\delta + \lambda_0} - b + \frac{\lambda_0 m}{\delta + \lambda_0}\left[p_1 - \frac{\delta(1-q)}{\delta + \lambda_1 q}(p_1 - p_2)\right]$$

so S^{**} increases in λ_1 and in q. This makes sense. If λ_1 is large then workers move quickly from type-p, jobs to type-p, jobs. In the words of Acemoglu and Shimer (1997), equilibrium is more efficient because workers can achieve a more efficient allocation by sampling more firms. If q is large then there are not many type-p, jobs in the first place.

3.4 More than two firm types

Conceptually, it is straightforward to generalize the above analysis to a finite number of firm types that is larger than two. For any possible lowest productivity level among active firms, it has to be checked whether the corresponding reservation wage is *both* below that level ***and*** above the highest productivity level among the non-active firms. However, if we increase the numbers of firm types, the parameter inequality that characterizes whether an equilibrium exists with all possible firm types becomes extremely cumbersome. In Appendix 1 we present, without proof, the full characterization of the equilibria in the model with three possible firm types. In that model, the three productivity values are taken to satisfy $0 \leq p_3 < p_2 < p_1 < \infty$ and $p_1 > b$. We denote the fraction of type-p, firms among the total measure of firms (active or non-active) by q_i, with $0 < q_i < 1$ and $q_1 + q_2 + q_3 = 1$. In this model, there are three possible equilibria, corresponding to whether $\phi < p_3$, or $p_3 \leq \phi < p_2$, or $p_2 \leq \phi$. In the second case, a fraction $q_1/(q_1 + q_2)$ (a fraction $q_2/(q_1 + q_2)$) of active firms has productivity p_1 (has productivity p_2).

Example 3. For a number of different labor markets in Denmark, Bunzel et al. (1999) (see Example 2) estimate the distribution of p to have three mass points. This includes the market for women aged below 22 with less than high school, the market for women aged above 50 with high school, the market for individuals aged between 31 and 50 with a bachelor degree, and the market for individuals aged between 22 and 50 with a master degree and higher. Using the results in Appendix 1 it can be shown that in all these labor markets, there exist exactly two possible equilibria. There is an equilibrium in which all three firm types are active (which is the equilibrium that is identified by the data), and there is an equilibrium in which only type-p, firms are active. The reservation wage in case of only type-p, firms and type-p, firms is below p_3, so there is no equilibrium with only type-p, firms and type-p, firms.

Example 4. Bowlus (1997) estimates equilibrium search models for a number of different US. labor markets, assuming that p has a discrete distribution. The setup is similar to Bunzel et al. (1999). 0 ne of the labor markets concerns full-time jobs for male college graduates who were aged between 14 and 22 in 1979 and who are followed until 1991. The number of firm types is estimated to equal 3. The other reported structural parameter estimates are $\lambda_0 = 0.031$, $\lambda_1 = 0.0064, \delta = 0.0023, p_3 = 607, p_2 = 804, p_1 = 1053, q_2 = 0.28, q_1 = 0.08$ (in U.S. Dollars and weeks).[12] In addition, ϕ, $E_G(w)$, and $\overline{w}$ are estimated to be 152, 564, and 807, respectively. From equations (1) and (2) it follows that $(\delta + \lambda_0)\phi = (\delta + \lambda_1)b + (\lambda_0 - \lambda_1)E_G(w)$, and this can be used to obtain the estimate that b = -998. The equilibrium with 3 firm types is unique. However, it is clear that the estimated value of b is implausibly low. This is may be a consequence of measurement errors in the wage data. If the true lowest wage in the market is higher than the observed lowest wage, and if this is ignored, then ϕ and b are under-estimated.[13] Suppose that the true value of b is 60 Dollars per week (instead of -998; it then follows that in the equilibrium with all three firm types the reservation wage of the unemployed young male college graduates is 431 Dollars per week instead of 152). Then there are two possible equilibria: one in which all three firm types are active, and one in which only type-pi and type-p_2 firms are active.

The labor markets in Examples 2-4 may display additional equilibria, in which currently latent firms are active. Those firms have productivity levels that are below the current reservation wage or mandatory minimum wage. It cannot be inferred empirically whether additional equilibria exist or what productivity levels those firms have, simply because we do not observe any activity at currently latent firms.

Now suppose that the distribution of p is continuously distributed across firms.

[12]The model estimated by Bowlus (1997) also allows for transition rates into and from non-participation. For our purposes, we may proceed by simply adding the value of the transition rate to non-participation η_1 to the value of 6. The former is so small for this market that this does not affect the first four digits of the value of δ.

[13]Of course in that case, the other estimates may be biased as well. Notably, if the model tries to fit an'empirical wage data distribution that has a fatter left-hand tail than it should have, then λ_1 may be under-estimated.

We denote the distribution function of p across all active and non-active firms by $\Gamma_0(p)$. This is a structural determinant. For convenience we assume that Γ_0 has a positive and continuously differentiable density γ_0 on the support $(\underline{p}_0, \overline{p})$, with $0 \leq \underline{p}_0 < \overline{p} \leq \infty$. We assume that $\Gamma_0(b) < 1$, which is a necessary condition to have production.

Let $\underline{p}$ denote the infimum productivity of firms which make a profit and thus are active on the market. This is of course not a structural determinant. The measure of active firms n_a equals $(1 - \Gamma_0(\underline{p}))n$. Let I'(p) denote the distribution of p among active firms,

$$\Gamma(p) = \frac{\Gamma_0(p) - \Gamma_0(\underline{p})}{1 - \Gamma_0(\underline{p})} \tag{17}$$

with $p \geq \underline{p}$.

Bontemps, Robin, and Van den Berg (2000) provide a comprehensive theoretical and empirical analysis of the model with a general continuous distribution for p. They show that $\underline{p} = \max\{\underline{p}_0, \phi\}$, that $\underline{w} = \phi$, and that $\phi < \overline{p}$, under the assumption that p has a finite mean, i.e. $E_{\Gamma_0}(p) < \infty$. Somewhat loosely, one may state that, in case of a continuous productivity distribution, only one wage is profit maximizing for a firm of a given type p. This defines the mapping $w = K(p)$. In equilibrium, K is increasing and non-linear in p and dependent all structural determinants.[14] The distribution of wage offers is F(w) $= \Gamma(K^{-1}(w))$. In general, no explicit expressions exist for equilibrium solutions.

Figure 2 displays the admissible equilibrium combinations of ϕ and the lowest productivity $\underline{p}$ among active firms in the market. A value $p > \underline{p}_0$ can only be the lowest p among active firms if it equals ϕ. If $\underline{p}_0$ is the low&t p among profitable firms then all values of ϕ below $\underline{p}_0$ are feasible.[15]

Using numerical examples, Bontemps, Robin, and Van den Berg (2000) show that for certain structural parameter values there is a unique equilibrium, while for others there is no equilibrium, and for yet others there are multiple equilibria.

[14]For a given value of ϕ, and with $\overline{\Gamma} := 1 -$ I', the equilibrium K satisfies
$w = K(p) = p - (\delta + \lambda_1\overline{\Gamma}(p))^2 \left[(\underline{p} - \phi)/(\delta + \lambda_1)^2 + \int_{\underline{p}}^{p} (\delta + \lambda_1\overline{\Gamma}(x))^{-2} dx\right]$ for all $p \in (\underline{p}, \overline{p})$.

[15]Suppose we would assume that the reservation wage of the unemployed has a fixed given value, and suppose we would solve the equilibrium given this value of ϕ. This gives outcomes for K, F, G etc. given ϕ. By substituting this G into equation (1) we obtain another value of ϕ, namely the value that is optimal given this G. We may do this for different starting values of ϕ, and the intersections of the resulting continuous curve with the diagonal (45”) line give the equilibrium values of ϕ. For a given equilibrium value of ϕ, the corresponding value of $\underline{p}$ can be derived in Figure 2 by going from the vertical axis to the horizontal axis using the solid curve. Note that the same procedure can be performed with Figure 1.

Given the results in the previous subsections, it is intuitively plausible that we may get multiple equilibria. The effect of the productivity distribution on the reservation wage works by way of the expectation of a monotone transformation of p, and a discrete distribution with a finite number of points of support can be approximated well by a continuous distribution, in terms of expectations.

Obviously, if $\lambda_0 = \lambda_1$ then there is a unique equilibrium. Bontemps, Robin, and Van den Berg (2000) also prove that there always exists an equilibrium if $\overline{p} < \infty$, provided that there is a mandatory minimum wage.[16] The type of non-existence of equilibrium that occurs if p is discrete with a finite number of points and $\lambda_0 < \lambda_1$ does not carry over to the model with continuous p, because, basically, the distribution I'(p) is now continuous in p. However, if $\overline{p} = \infty$ then there is a different type of non-existence of equilibrium. Somewhat loosely, if ϕ increases then the distribution of p among active firms may become much more attractive, pushing up ϕ even more, etc.

In Appendix 2 we prove the following additional result:

If $\lambda_0 > \lambda_1$, and if the continuous distribution Γ_0 has an increasing failure rate, and $\underline{p}_0 \leq b$, then there is a unique equilibrium.

The failure rate (or hazard rate) of the distribution Γ_0 is defined as $\gamma_0(p)/\overline{\Gamma}_0(p)$, with $\overline{\Gamma}_0 = 1 - \Gamma_0$. The distribution has an increasing failure rate (IFR) if the derivative of the failure rate is nonnegative for all $p \in (\underline{p}_0, \overline{p})$. IFR is equivalent to log concavity of $\overline{\Gamma}_0$ and is implied by log concavity of γ_0.

Somewhat loosely, one may say that IFR implies that the right-hand tail of Γ_0 should be *at least as thin* as the tail of an exponential distribution. Examples of IFR distributions of nonnegative random variables are exponential distributions and normal distributions truncated from below. For other examples and a survey of these concepts, see Van den Berg (1994). Unfortunately, IFR is an untenably strong assumption for productivity distributions. Bontemps, Robin and Van den Berg (2000) show that the equilibrium Γ_0 should actually have a very *fat* right-hand tail if the model is to give an accurate fit to wage data. Intuitively, this may be plausible, as the productivity distribution dominates the wage distribution, and there is abundant empirical evidence that wage (offer) distributions have fatter right-hand tails than as required by IFR (see Van den Berg, 1994). In fact, Bontemps, Robin and Van den Berg (2000) provide direct firm-data evidence that the distribution of p has a fat tail and is not IFR (see Forslund and Lindh, 1997, for similar evidence). Therefore, the above uniqueness result is of limited

[16] The condition that there is a mandatory minimum wage is not necessary.

usefulness.

4 Endogenous job offer arrival rates

As mentioned in the introduction, it could be argued that λ_0 and λ_1 may depend on n_a. This would have implications for the analysis above, since n_a is smaller in the equilibrium with only type-p, firms than in the other equilibrium. The way in which λ_0 and λ_1 depend on n_a follows from the matching technology on the labor market and the search behavior of workers and firms. If n_a decreases then, on the one hand, there are less firms to be sampled by workers. On the other hand, the remaining firms will be larger (they will have larger labor forces) so they may well be easier to locate by searching workers. If these effects counterbalance then λ_0 and λ_1 are independent of n_a, and the results of the previous section follow.

In this section we adopt a more general approach. Suppose that λ_0 and λ_1 are both functions of n_a, but that there is a constant α such that always $\lambda_1(n_a)/\lambda_0(n_a) = \alpha$ (below we provide a justification for such a specification in terms of a matching function). Since we are interested in the extent to which the previous results on multiplicity of equilibria carry through, we assume in this section that $0 < \alpha < 1$ (so $\lambda_0 > \lambda_1$). From Proposition **1,** the equilibrium in which only type-p_1 firms are active exists if and only if $\alpha(1 - \alpha)(\lambda_0(qn))^2(p_1 - p_2) \geq (\delta + \alpha\lambda_0(qn))^2(p_2 -$ b). From Proposition 2, the equilibrium in which both types of firms are active exists if and only if $\alpha(1 - \alpha)(\lambda_0(n))^2q^2(p_1 - p_2) \leq (\delta + \alpha\lambda_0(n)q)^2(p_2 -$ b). Thus, these two equilibria both exist if and only if

$$\frac{\alpha(1-\alpha)(\lambda_0(qn))^2}{(\delta+\alpha\lambda_0(qn))^2} \geq \frac{p_2-b}{p_1-p_2} > \frac{\alpha(1-\alpha)(\lambda_0(n))^2q^2}{(\delta+\alpha\lambda_0(n)q)^2} \tag{18}$$

(Note that at this stage we cannot preclude existence of more equilibria.) The inequalities above imply the following,

Proposition 4 *For any δ and α there are values of p,, p_2 and b for which both equilibria exist, if and only if the following inequality holds,*

$$\lambda_0(qn) > q\lambda_0(n) \tag{19}$$

Intuitively, it may seem that this condition is always satisfied if the matching technology displays decreasing returns to scale in the number of searching individuals, which is a fairly innocuous assumption (for example, this follows if the matching rate displays constant returns to scale in the number of searching individuals and the number of vacancies taken together). However, the situation is somewhat more complicated than this, because now the unemployment rate also depends on n_a, and unemployed and employed searchers have different search intensities.

To proceed, we assume that the flow of contacts between workers and employers is a function $M(\cdot,\cdot)$ of the "effective" measure of searching workers and the measure of vacancies. In line with the literature, this function is called the matching function (note that here "contact function" would be a more appropriate name). First consider the second argument of $M(\cdot,\cdot)$. Note that all active firms always want to expand, as the profit per additional worker is always strictly positive. These firms always have a vacancy, and they wait passively for searching workers. Thus, the measure of vacancies equals the measure n_a of active firms. Now let us turn to the first argument of $M(\cdot,\cdot)$. All workers always search for (better) jobs. The "effective" measure of searching workers may differ from the measure of searching workers m because it takes into account that some workers may have different search intensities than others. We assume that workers' search intensities are always at their physical maximum, which depends on the worker's labor market state but does not vary across workers. Let $\alpha \in (0,1)$ denote the relative search efficiency of search by employed workers in comparison to unemployed workers. Then the effective measure of searching workers can be written as $u + \alpha(m - u)$. It follows that

$$\lambda_0(n_a) = \frac{M(u+\alpha(m-u), n_a)}{u+\alpha(m-u)}, \quad \lambda_1(n_a) = \frac{M(u+\alpha(m-u), n_a)}{u+\alpha(m-u)} \cdot \alpha \qquad (20)$$

so that indeed $\alpha \equiv$ X,/X,.

We have to check whether the inequality (19) holds. To proceed, we assume that $M(\cdot,\cdot)$ displays constant returns to scale (CRS), which is in line with the empirical literature on matching functions for the labor market (see e.g. Mortensen and Pissarides, 1999). In particular, we adopt a CRS Cobb-Douglas specification,

$$M(u+\alpha(m-u), n_a) = \mu \cdot (u+\alpha(m-u))^{1-\beta} n_a^{\beta} \qquad (21)$$

For ease of exposition we rule out that $\beta = 1$, so that $0 \leq \beta < 1$. Note that $\beta = 0$ produces the model of the previous section.

It is useful to start assuming (incorrectly) that (20) and (21) determine how λ_0 changes in response to n_a (so that u is a fixed constant). It is straightforward to see that λ_0 then increases with n_a, but that the increase is sufficiently modest for inequality (19) to hold for any $0 < q < 1$ (this does not depend on the Cobb-Douglas specification).

In reality, u also changes with n_a. From Section 2,

$$\frac{u(n_a)}{m} = \frac{\delta}{\delta + \lambda_0(n_a)} \tag{22}$$

By substituting (22) into (20), using (21), we obtain a relation that can be written as the inverse of the function $\lambda_0(n_a)$ if $\beta > 0$,

$$n_a = \lambda_0^{\frac{1}{\beta}} \mu^{\frac{1}{\beta}} m \left[\alpha + (1-\alpha)\frac{\delta}{\delta + \lambda_0}\right] \tag{23}$$

It can be shown that this function is increasing,[17] so λ_0 increases in n_a. From (22), this implies that u decreases with n_a. This in turn implies that λ_0 increases stronger with n_a than if u were a fixed constant. In words, an increase in the number of active firms increases the job offer arrival rate of the unemployed, which in turn decreases unemployment, and this gives a further boost to the arrival rate of the remaining unemployed.

It remains to examine under which conditions λ_0 does not increase so strongly with n_a that **(19)** does not hold anymore. For reasons of continuity it is clear that if $u(n_a)$ is rather insensitive to n_a (e.g. because β is small) or if $M(.,.)$ is rather insensitive to its second argument (e.g. because β is small) then **(19)** still holds. More generally, we have

Proposition 5 *If the parameter β of the matching function satisfies*

$$0 \le \beta \le \frac{1}{2} + \frac{1}{2}\sqrt{\alpha}$$

then for any δ and for any $q \in (0,1)$ one can always find values of p_1, p_2 and b such that both equilibria exist.

The proof is in Appendix 3. The condition on β is quite weak. For moderate values of α, the upper bound exceeds 0.75, which is larger than typical estimates of β. In Example 1 in the previous section, $\alpha = 0.33$, which corresponds to $\beta < 0.79$. In fact, the above sufficient condition on β is by no means necessary. In Example 1, the inequality $\lambda_0(qn) > q\lambda_0(n)$ holds for all q for values of β as high as 0.82. In general, though, if β is much higher than $(1 + \sqrt{\alpha})/2$, and is very close to 1, then this inequality does not hold for any q. It should also be noted

[17] The derivative of n_a with respect to λ_0 is proportional to $x^{\frac{1}{\beta}-1}(1+x)^{-2}\left[\alpha x^2 + (1-\alpha)(1-\beta)x + 1\right]$, with $x = \lambda_0/\delta$. The term in square brackets is positive for all $x \geq 0$.

that if α is close to **1** then the range of values of p_1, p_2 and b for which multiple equilibria exist is very narrow.

Now let us turn to the social welfare flows. The entities S* and S** are defined as in the previous section, the only difference being that S^* now is a function of qn, since it depends on $\lambda_0(qn)$. In comparison to the previous section, S* is now lower. If the arrival rates depend on the measure of active firms then the arrival rates in the equilibrium with only type-pi firms are lower than before. This increases the unemployment rate, which in turn decreases social welfare. As a result, the difference between S* and S** is now smaller than before,

$$\frac{\delta + \lambda_0(n)}{\delta m}\left(S^*(qn) - S^{**}\right) = \frac{\lambda_0(n)(1-q)}{\delta + \alpha\lambda_0(n)q}(p_1 - p_2) - \frac{p_1 - b}{\delta + \lambda_0(qn)}(\lambda_0(n) - \lambda_0(qn))$$

The first term on the right-hand side represents the familiar productivity gain for the type-p, equilibrium. The second term represents the loss due to increased unemployment. Under rather extreme conditions, the sum can be negative. There do not seem to be transparent conditions on the structural parameters under which the sum is always positive?

Finally, we return to the examples discussed in the previous section. In these examples, p_1, p_2 and b have fixed values, so that the above proposition cannot be applied. We can however check for which values of $\lambda_0(qn)$ (or for which values of β) there are multiple equilibria, by examining the inequalities (18). The values of the job offer arrival rates given in the previous section correspond to the equilibrium in which both types of firms are active, so the second inequality in **(18)** is satisfied and this equilibrium definitely exists. For the other equilibrium to exist, the first inequality in **(18)** has to hold. This inequality can be rewritten as an inequality on $\lambda_0(qn)$,

$$\lambda_0(qn) > \frac{\delta}{-\alpha + \sqrt{\frac{\alpha(1-\alpha)(p_1-p_2)}{p_2-b}}}$$

Example 1 (continued). Recall that $\lambda_0(n) = 0.15$ and that 25% of the firms have a high productivity. With both types of firms active, the mean social welfare S^{**}/m equals 1156. According to the above inequality, the equilibrium with only high productivity firms exists if $\lambda_0(qn) > 0.072$. So, in those cases there are

[18] Swinnerton (1996) examines an equilibrium search model with wage setting, decreasing returns to scale in production, and absence of on-the-job search. The equilibrium is unique, but minimum wage increases may result in more rejection of applicants by firms as well as higher production efficiency. As in our model, the net effects on welfare as well as unemployment may be positive at the same time.

two equilibria. Consider the equilibrium with only type-p, firms. Within the range of admissible values of $\lambda_0\ (qn)$, the lowest social welfare and the highest unemployment rate are attained if $\lambda_0(qn) = 0.072$. This gives $u/m = 0.12$ and $\mathcal{S}^*(qn)/m = 1176$. This social welfare is still higher than $\mathcal{S}^{**}$ despite the fact that unemployment has almost doubled. If $\lambda_0(qn) > 0.072$ then $S^*(qn)$ is even higher; if $\lambda_0(qn) = \lambda_0(n) = 0.15$ then $\mathcal{S}^*(qn)/m = 1188$. The values of β that give matching functions which generate $\lambda_0(qn) \geq 0.072$ can be determined by evaluating (20) and (21) at qn and n. As a result, $0 \leq \beta \leq 0.50$.

Example 2 *(continued).* Recall that here $\lambda_0(n) = 0.034$ and that about 13% of the firms have a high productivity. $\mathcal{S}^{**}/m$ equals 293. The equilibrium with only high productivity firms exists if $\lambda_0(qn) \geq 0.021$. If $\lambda_0(qn) = 0.021$ then the equilibrium with only type-p, firms has $u/m = 0.38$ and $\mathcal{S}^*(qn)/m = 753$. Thus, social welfare is again still higher than in the other equilibrium. (If $\lambda_0(qn) = 0.034$ then $\mathcal{S}^*(qn)/m = 884$.) The values of β in $[0, 0.22]$ give matching functions which generate $\lambda_0(qn) \geq 0.021$.

5 A minimum wage

In this section we focus on markets with multiple equilibria, that is, we assume that (16) holds, which implies that $\lambda_0 > \lambda_1$ and $b < p_,$. Let ϕ^* and ϕ^{**} denote the reservation wages in the equilibrium in which only type-p, firms are active and in the other equilibrium, respectively. Now suppose that the labor market is in the equilibrium in which both types of firms are active, and suppose that a mandatory minimum wage w_L is imposed. We assume full coverage of this minimum wage. We can distinguish between five cases depending on the relative value of w_L.

(A). If $w_L \leq \phi^{**}$ then this does not have any effect on the equilibrium outcome.

(B). If $\phi^{**} < w_L < p_2$ then the equilibrium is still such that both types of firms are active. However, w_L replaces ϕ^{**} as the lowest wage in the market. The market power of workers increases at the expense of the firms' monopsony power. As a result, the whole wage (offer) distribution shifts upwards. This has a positive effect of the reservation wage ϕ^{**}. At first sight one may think that the new ϕ^{**} may exceed w_L. However, this is not possible as it would imply that

there are two different equilibria in which both types of firms are active.[19] The equilibrium outcomes (10)–(13) are still valid, provided that ϕ is replaced by w_L.

(C). If $p_2 \leq w_L < \phi^*$ then type-p, firms cannot operate profitably any-more, and the unfavorable equilibrium is replaced by the favorable equilibrium?' However, the outcome of the latter equilibrium is not affected in any way by w_L since it is smaller than ϕ^*. As a result, the imposition of w_L induces a shift in the equilibrium, but the value of w_L itself does not enter the new equilibrium outcomes. The minimum wage is strictly smaller than the lowest wage in the market, and consequently the wage (offer) density does not display a spike at it.

(D). If $\phi^* \leq w_L < p_1$ then again the unfavorable equilibrium is replaced by the favorable equilibrium, but now w_L affects the outcome of the latter equilibrium since it is the lowest wage in the market. The outcome in case of a homogeneous model with $\underline{w} = w_L$ has been analyzed extensively in Van den Berg and Ridder (1998).[21]

(E). If $w_L \geq p_1$ then both type-p, and type-p, firms cannot operate profitably, and there will not be any production.

It is also interesting to examine the effect of a subsequent abolition of the minimum wage. For this we need to be more specific about the determinants of market entry by currently latent firms. We simply assume that, if active firms are already present, then the entry costs faced by the latent firms are so large that there is no entry even if the expected instantaneous profit flow of the latent firm becomes positive due to the abolition of the minimum wage. This is of course unlikely as a long-run outcome, but it could be regarded as a medium-run outcome. A more sophisticated analysis would be extremely difficult, as it would also have to deal with out-of-equilibrium dynamics.

[19] The expression for ϕ^{**} in case $\phi^{**} < w_L < p_2$ is as follows,

$$\phi^{**} = \frac{(\delta+\lambda_1)^2 b + (\lambda_0 - \lambda_1)\delta w_L + (\lambda_0 - \lambda_1)\lambda_1(cp_1 + (1-c)p_2)}{(\delta+\lambda_1)(\delta+\lambda_0)}$$

with c as in (14). The expression on the right-hand side is smaller than w_L iff the expression on the right-hand side of (14) is smaller than w_L.

[20] In practice, the minimum wage level may not be determined by maximization of social welfare. Sobel (1999) convincingly argues that the relative strength of interest groups in the political process is an important determinant.

[21] The expression for ϕ^* in case $\phi^* \leq w_L < p_1$ is as follows,

$$\phi^* = \frac{(\delta+\lambda_1)^2 b + (\lambda_0 - \lambda_1)\delta w_L + (\lambda_0 - \lambda_1)\lambda_1 p_1}{(\delta+\lambda_1)(\delta+\lambda_0)}$$

The expression on the right-hand side is smaller than w_L iff the expression on the right-hand side of (8) is smaller than w_L.

If w_L is abolished in *(A)*, then nothing changes. In *(B)*, *(D)* and *(E)*, the equilibrium outcomes do change. In *(B)* and *(D)*, the set of active firms does not change, but the wage distribution shifts towards lower wages. The equilibrium in *(B)* returns to the equilibrium with outcomes that are equal to those before imposition of w_L. In *(C)*, the equilibrium outcome does not change, meaning that the temporary imposition of w_L has established a permanent shift from the unfavorable to the favorable equilibrium.

Let us return to the effects of the imposition of w_L. Obviously, the most interesting cases above are *(C)* and *(D)*, because then the minimum wage induces a shift to the more favorable equilibrium. Case *(C)* is the most intriguing. From the fact that the minimum wage is strictly smaller than the lowest wage in the market, a casual observer may induce that the minimum wage is not "binding" and is thus irrelevant. Indeed, he may be strengthened in this belief in case of an abolition of the minimum wage, since the latter does not affect equilibrium.

It is important to stress that these results are not due to the assumption that p can only attain two values. Recall from Subsection 3.4 that multiple equilibria are possible if p has a continuous distribution or a discrete distribution with more than two points of support. In the model with a continuously distributed p, the imposition of a minimum wage in between the two reservation wages that are associated with the two equilibria ensures that one of the equilibria cannot exist anymore, but it does not affect the outcome of the other equilibrium. This corresponds to case *(C)* above.

The minimum wage effects are consistent with a number of stylized facts (see Card and Krueger, 1995). First, in general, an increase in the minimum wage decreases wage variation. Secondly, an increase in the minimum wage does not have a large effect on employment. In the model versions of Section 3, the employment effect is zero as long as the minimum wage does not cross the highest p in the market. In the model of Section 4 the employment effect is zero as long as the minimum wage does not cross p_2. If it does, then the magnitude of the negative employment effect depends on the shape of the matching function. Thirdly, an increase in the minimum wage has a positive effect on wages above it.

It is often argued that a spike in the wage density at the minimum wage indicates that the minimum wage has an effect on the wage density, whereas the absence of such a spike indicates that the minimum wage is irrelevant. Clearly, the results above question the universal validity of such a view. It should be stressed that our model does not preclude the presence of a spike at w_L. In case *(B)* above, the wage (offer) density displays a spike at w_L if the latter is only marginally lower than p_2 . In the latter case, all type-p, firms are active, and they are all forced to pay a wage in between w_L and p_2. A likewise situation occurs in case *(D)* above, if w_L is only marginally lower than p_1. Bontemps, Robin and Van den Berg (2000) show that such a "congestion" spike in the wage density at w_L can also be generated if p is continuously distributed. Interestingly, wage data from European countries often do not display a spike at the minimum wage even though there is typically full compliance (see evidence in Östros, 1994, Koning, Ridder and Van den Berg, 1995, Van den Berg and Ridder, 1998, Bunzel et al., 1999, and Bontemps, Robin and Van den Berg, 2000).

Now let us turn to some practical issues that come up if one would use the above insights for policy. First of all, in reality, different individuals have different individual-specific productivity components. The model framework does not allow for dispersion of such components within a labor market. Therefore, the results seem to be more relevant for labor markets with a homogeneous kind of work, where individual-specific productivity differences are relatively small. Secondly, recall that firms are assumed to set wages rather than bargain over wages with workers. The theoretical studies by Manning (1993) and Ellingsen and Rosen (1997) examine under which conditions a firm is more likely to set wages rather than bargain over them. Both conclude that wage setting is more relevant for jobs that require less skills. Thirdly, a minimum wage is obviously a more relevant policy instrument for labor markets with relatively low productivity levels. In sum, the results in the present paper seem to be particularly relevant for labor market segments at the bottom of the economy, with jobs that do not require high skills.

Suppose that the labor market consists of a number of separate labor markets called segments. Each segment is a separate labor market of its own, and workers in a particular segment are homogeneous. Within a segment, labor productivity may be dispersed among the firms that are active in the segment. In each segment, the equilibrium then is as in our model. However, segments may differ in terms of their individual-specific productivity, so the distribution of p may vary across segments. In that case, the imposition of a minimum wage that exceeds the highest possible productivity level within a particular segment causes all firms in that segment to become unprofitable. All individuals associated with that segment then become permanently (or structurally) unemployed. Let the distribution of p_1 among the population of individuals be denoted by H. (For example, if there are two segments, one with 75% of all individuals and with $p_1 = 1200$ and one with 25% of all individuals and with $p_1 = 1500$ then $H(x) = 0.75$ for

$x \in [1200, 1500)$.) Then the unemployment rate is equal to

$$\frac{u}{m} = \frac{\delta}{\delta + \lambda_0}(1 - H(w_L)) + H(w_L)$$

The first and second term on the right-hand side reflect frictional unemployment and structural unemployment, respectively. Here, we assume that a single value of w_L holds for all segments.[22] In the current setup it is however optimal to have a different minimum wage for each segment. This of course requires knowledge of the highest productivity value in each segment. In many European countries, trade unions and employer federations meet every year to negotiate the minimum wage for different segments (i.e. different occupations within different sectors) of the labor market. Such negotiated segment-specific minimum wages are often subsequently legally imposed by the government. The analysis in this section provides a justification of this practice.

6 Heterogeneous unemployed workers

So far we have assumed that workers are homogeneous, and, in particular, that the opportunity value of employment b is a fixed constant. This assumption is violated if some workers enjoy leisure more than others, of if unemployment benefits are dispersed. In this section we show that the main results above are robust with respect to this. We analyze an equilibrium search model in which workers on the same labor market can have different individual-specific b and firms can have different production technologies. For expositional reasons we now abstract from search on the job. Furthermore, we simply assume that the b can attain two possible values (b_1 and b_2, with probabilities π and $1 - \pi$)), and (again) that p can attain two possible values (p_1 and p_2 with probabilities q and $1 - q$). The relevant theoretical literature includes MacMinn (1980) and Albrecht and Axe11 (1984), who actually assume a continuous distribution of p across firms. The Albrecht and Axell (1984) model is estimated by Eckstein and Wolpin (1990). Mortensen (1990) derives a number of properties of equilibria in a model with discrete b and p.

If unemployed workers have different b then in general they also have different reservation wages. The most important equilibrium property is that the support of the equilibrium wage offer distribution is a subset of (or coincides with) the set of unemployed workers' reservation wages. Basically, any other offer value

[22] If H is continuous, and w_L is in its support, then typically the value of the population wage density at w_L^+ is positive.

can be improved upon by offering the reservation wage below the offer: this gives a higher profit per worker without any loss in the size of the labor force of the firm. Another equilibrium property is that higher-productivity firms offer higher wages and are therefore able to attract more workers.

We assume that $b_2 < p_2 < b_1 < p_1$. Note from equation (1) that a worker's reservation wage ϕ_i is now always larger than or equal to his benefits level b_i (take $\lambda_1 = 0$; they are equal iff $b_i = \overline{w}$). Equilibrium is either such that $\phi_2 < p_2$, or such that $\phi_2 \geq p_2$. In the first case, all firms are active. Type-p_2 firms offer ϕ_2, and type-p_1 firms may offer ϕ_1 and/or ϕ_2, depending on the parameter values. In the second case, only type-p_1 firms are active, and they may offer ϕ_1 and/or ϕ_2. In Appendix 4 we prove the following result.

Proposition 6 *In the model with heterogeneity of workers' unemployment benefits, assume that*

$$\pi(\delta + \lambda_0)(p_1 - b_2) > (\delta + \pi\lambda_0)(b_1 - b_2) \tag{24}$$

There exist exactly two equilibria if and only if

$$\frac{\delta}{\lambda_0 q} > \frac{b_1 - p_2}{p_2 - b_2} \geq \frac{\delta}{\lambda_0} \tag{25}$$

In one equilibrium, only type-p_1 firms are active. In the other, both types are active. If (25) is not satisfied then there exists exactly one equilibrium.

(Note that p_1 and π do not affect (25), whereas p_2 does not affect (24), so that (24) cannot be in conflict with (25).)

The mechanism behind the multiplicity of equilibrium is essentially the same as in Section 3. In one of the equilibria, both high-productivity and low-productivity firms are active. In that equilibrium, the former firms offer b_1 and the latter firms offer ϕ_2, which in turn has a rather low value. In the other equilibrium, only high-productivity firms are active, and they offer b_1. The unemployed workers with a low benefits level then have a rather high reservation wage ϕ_2. Indeed, in this second equilibrium, $\phi_2 > p_2$, so ϕ_2 acts as a binding lower bound on the set of production technologies that enable a positive profit per worker. It rules out production at the low productivity level. The resulting wages are higher than in the first equilibrium. This in turn justifies the higher reservation wage in the second equilibrium. Thus, like in Section 3, the reservation wage of the unemployed affects the set of profitable production technologies, while the set of production technologies in use in turn affects the reservation wage, and this system has multiple solutions.

Analogous to Section 4, one may argue that λ_0 may depend on the measure of active firms n_a. However, it is easy to see from (25) that, like in Section 4, the condition (19) on $\lambda_0(qn)$ is necessary and sufficient for both equilibria to be possible. Specifically, if (24) is satisfied then there are values of p_2, b_1 and b_2 for which both equilibria exist, if and only if $\lambda_0(qn) > q\lambda_0(n)$.

The analysis of minimum wage effects is also essentially the same as before. That is, one may single out the equilibrium with only type-p, firms by imposing a minimum wage w_L satisfying $p_2 \leq w_L \leq p_,$. If this w_L is smaller than b_1 then it is strictly smaller than the new lowest wage in the market. In comparison to the previous sections, the model with heterogeneous b provides an additional reason for social welfare to be higher in case of the equilibrium with only type-p, firms (i.e. in case of a minimum wage exceeding p_2). This reason is nothing but the traditional argument for a minimum wage in monopsonistic labor markets: it increases employment (see Albrecht and Axell, 1984, and Eckstein and Wolpin, 1990). In the equilibrium in which both types of firms are active, type-b, individuals reject job offers from type-p, firms. However, in the other equilibrium, all job offers are always accepted, so unemployment is lower.

Note that imposition of a relatively high minimum wage, as a means to single out the desirable equilibrium, should be preferred over an over-all increase of unemployment benefits with an amount A $\geq p_2 - b_2$. The latter policy also ensures that type-p, firms are not active; however, the new benefits level b_1 + A of the type-b, individuals may be larger than p_1, in which case these individuals will become permanently unemployed.

In The Netherlands, as in many other countries, the level of social assistance benefits for unemployed workers is mostly determined by household composition (see Van den Berg, Van der Klaauw and Van Ours, 1998, for details; unemployed workers may be entitled to such benefits if their unemployment insurance entitlement has expired). The model in this section has some relevance for the labor market of such workers. Imposition of a minimum wage just below the highest possible social assistance level has the advantages that (1) the equilibrium is such that workers only work at higher-productivity firms and (2) frictional unemployment is reduced.

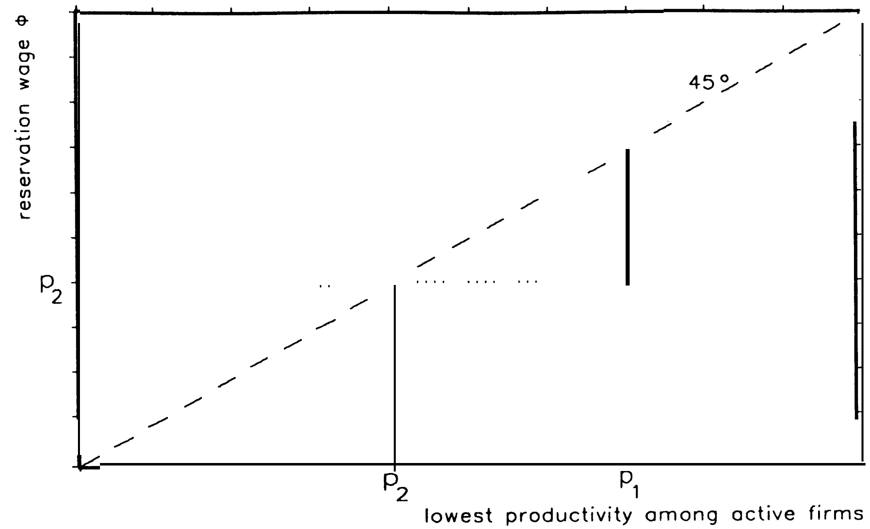
7 Conclusion

In this paper we have examined the mutual dependence of the reservation wage of the unemployed and the minimum productivity level at which production is profitable. Specifically, we showed that this dependence can generate multiplicity of equilibrium. Using results from the literature on structural estimation of equilibrium search models, we showed that multiplicity is an empirically relevant phenomenon. These results remain valid under a number of model extensions. In particular, we accounted for the fact that the workers' job offer arrival rates depend on the number of productive firms. In addition, we examined a model in which unemployed workers differ in terms of their unemployment benefits or value of leisure. In that case, the lowest reservation wage among the unemployed interacts with the lowest profitable productivity level.

These results imply that a minimum wage policy can be fruitfully applied to single out the desirable equilibrium. In such a case, the resulting minimum wage may wrongfully appear to be irrelevant, as its value can be strictly smaller than the lowest wage in the market. The results provide a justification of a policy in which minimum wages are sector-specific and occupation-specific.

FIGURE 1. Admissible equilibrium combinations

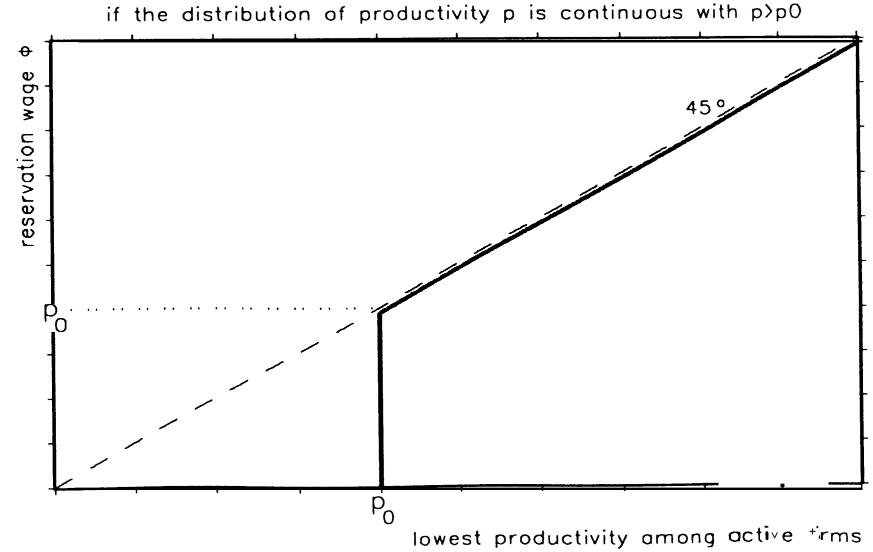
if the distribution of productivity p is discrete at p1 and p2



Note: the solid lines give the admissible combinations of reservation wage and lowest productivity among active firms.

FIGURE 2. Admissible equilibrium combinations

if the distribution of productivity p is continuous with p>p0



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Appendix

Appendix 1. Three firm types.

There is an equilibrium in which only type-p, firms are active if and only if

$$\frac{(x,-x,)x,}{(\delta+\lambda_1)^2} > \frac{p_2 - b}{P1 - p_2}$$

This is of course equivalent to Proposition **1.**

There is an equilibrium in which type-p, and type-p, firms are active but type-p, firms are not active if and only if

$$\frac{p_2-b}{p_1-p_2} > \frac{(\lambda_0-\lambda_1)\lambda_1}{(6+\lambda_1)^2}\, c \geq \frac{c(p_3-b)}{c(p_1-P2)+(p_2-p_3)} \tag{26}$$

with

$$c = \left(\frac{(\delta+\lambda_1)q}{\delta+\lambda_1 q}\right)^2, \qquad \text{with} \qquad q = \frac{q_1}{q_1+q_2}$$

Note that $0 < c < 1$. The first inequality in (26) is of course equivalent to Proposition 2.

Finally, there is an equilibrium in which all three firm types are active if and only if

$$\frac{(\lambda_0-\lambda_1)\lambda_1}{(\delta+\lambda_1)^2} < \frac{p_3-b}{c_1(p_1-p_2)+c_2(p_2-P3)}$$

with

$$c_1 = \left(\frac{(\delta+\lambda_1)q_1}{\delta+\lambda_1 q_1}\right)^2 \qquad \text{and} \qquad c_2 = \left(\frac{(\delta+\lambda_1)(q_1+q_2)}{\delta+\lambda_1(q_1+q_2)}\right)^2$$

so that $0 < c_1 < c_2 < 1$.

It cannot be ruled out a priori that there are multiple equilibria. Let, for a given equilibrium, the numbers in a set $\{\,.\,..\}$ indicate which firm types are active. It is possible that for given parameter values there exist $\{1,2,3\}$, $\{1,2\}$, and $\{1\}$ equilibria. It is also possible that for given parameter values there exist $\{1,2,3\}$ and $\{1\}$ equilibria but no $\{1,2\}$ equilibrium. It is also possible that for given parameter values there exist $\{1,2\}$ and $\{1\}$ equilibria but no $\{1,2,3\}$ equilibrium.

Appendix 2. Uniqueness in case of IFR distributions.

If $\lambda_0 > \lambda_1$ and $\underline{p}_0 \leq b$ then, by equation **(1),** any equilibrium solution for ϕ satisfies $\phi \geq \underline{p}_0$, which implies that any equilibrium solution for p satisfies $\underline{p} = \phi$. By substituting F(w) = K(p) into equation **(1),** using the expression for K(p) in footnote 14, and using the relation between Γ and Γ_0, and imposing that $p = \phi$, we obtain,

$$\phi = b + (\lambda_0 - \lambda_1) \int_\phi^{\overline{p}} \left[\frac{\overline{\Gamma}_0(p)}{\delta\overline{\Gamma}_0(\phi) + \lambda_1\overline{\Gamma}_0(p)}\right]^2 dp \tag{27}$$

Any equilibrium is characterized by a solution of this equation for ϕ. Consider the left-hand and right-hand sides as functions of ϕ on $(b,\overline{p})$. At ***b,*** the left-hand side is smaller than the right-hand side. The left-hand side is strictly increasing everywhere. The right-hand side is not decreasing everywhere for every possible Γ_0. To proceed, consider the derivative of the right-hand side with respect to ϕ. This equals

$$\frac{\lambda_0 - \lambda_1}{\delta}\left[\frac{-\lambda_1\delta}{(\delta+\lambda_1)^2} + \frac{\gamma_0(\phi)}{\overline{\Gamma}_0(\phi)} \cdot \int_\phi^{\overline{p}} s'\left(\frac{\overline{\Gamma}_0(p)}{\overline{\Gamma}_0(\phi)}\right) \cdot \frac{\overline{\Gamma}_0(p)}{\overline{\Gamma}_0(\phi)}\, dp\right] \tag{28}$$

with

$$s'(x) = \frac{2\lambda_1\delta^2 x}{(\delta + \lambda_1 x)^3}$$

We apply partial integration to (28), going from $-s'(\overline{\Gamma}_0(p)/\overline{\Gamma}_0(\phi))\gamma_0(p)/\overline{\Gamma}_0(\phi)$ to its integral $s(\overline{\Gamma}_0(p)/\overline{\Gamma}_0(\phi))$. The function s(x) equals

$$s(x) = \frac{\lambda_1\delta x^2}{(\delta + \lambda_1 x)^2},$$

In the process, we exploit the fact that the IFR property implies that $\gamma_0(p)/\overline{\Gamma}_0(p)$ is strictly larger than 0 for all p close to $\overline{p}$. After much rewriting, it follows that (28) equals

$$\frac{\lambda_0 - \lambda_1}{\delta}\frac{\gamma_0(\phi)}{\overline{\Gamma}_0(\phi)} \int_\phi^{\overline{p}} s\left(\frac{\overline{\Gamma}_0(p)}{\overline{\Gamma}_0(\phi)}\right) \cdot \frac{d}{dp}\left(\frac{\overline{\Gamma}_0(p)}{\gamma_0(p)}\right) dp$$

If Γ_0 is IFR then the derivative of $\overline{\Gamma}_0(p)/\gamma_0(p)$ is negative for all $p < \overline{p}$. Therefore, (28) is negative for all $\phi \in$ (b, $\overline{p}$). This implies that the right-hand side of (27) is decreasing everywhere. As a result, there is at most one solution to (27). If $\overline{p} < \infty$ then we already know from Bontemps, Robin and Van den Berg (2000) that there exists an equilibrium, so then the equilibrium is unique. If $\overline{p} = \infty$ then it follows directly from (27) that there is always a value of ϕ at which both sides intersect, so then the equilibrium is unique as well. $\square$

Appendix 3. Proof of Proposition 5.

The condition **(19)** holds for all $0 < q < 1$ iff the inverse function $n_a(\lambda_0)$ satisfies

$$\frac{n_a(\lambda_0)}{n} < \frac{\lambda_0}{\lambda_0(n)}.$$

for all $0 < \lambda_0 < \lambda_0(n)$, where λ_0 is the argument of the function $n_a(\,.)$, and n and $\lambda_0(n)$ are fixed constants. We first consider the case $\beta > 0$. Equation (23) can be rewritten as

$$\frac{n_a}{n} = \left(\frac{\lambda_0}{\lambda_0(n)}\right)^{\frac{1}{\beta}} \frac{\alpha + (1-\alpha)\frac{\delta}{\delta+\lambda_0}}{\alpha + (1-\alpha)\frac{\delta}{\delta+\lambda_0(n)}}.$$

Therefore, $n_a/n < \lambda_0/\lambda_0(n)$ iff

$$\frac{\lambda_0^{\frac{1}{\beta}-1}}{\delta+\lambda_0}(\delta + \alpha\lambda_0) < \frac{\lambda_0(n)^{\frac{1}{\beta}-1}}{\delta+\lambda_0(n)}(\delta + \alpha\lambda_0(n)) \tag{29}$$

We already know that $\lambda_0(n_a) < \lambda_0(n)$, so if the function $x^{\frac{1}{\beta}-1}(\delta+\alpha x)/(\delta+x)$ strictly increases in x for $x \in (0, \lambda_0(n)]$, then the inequality (29) is true for all $\lambda_0 \in (0, \lambda_0(n))$ (i.e. for all $q \in (0,1)$). The derivative of this function with respect to x is proportional to

$$\alpha(1-\beta)x^2 + \delta(1+\alpha-2\beta)x + \delta^2(1-\beta) \tag{30}$$

Clearly, the values of this expression are positive for every $x \geq 0$ if the second coefficient $\delta(1+\alpha-2\beta)$ is non-negative. This gives the sufficient condition $0 < \beta \leq (1+\alpha)/2$. Expression (30) is a second-degree polynomial in x. After some elaboration it follows that its values are positive on $\mathbb{R}$ iff $(1-\sqrt{\alpha})/2 < \beta < (1+\sqrt{\alpha})/2$, and its values are positive on $\mathbb{R}$ except at one point iff $\beta = (1-\sqrt{\alpha})/2$ or $\beta = (1+\sqrt{\alpha})/2$. This gives the second sufficient condition: $(1-\sqrt{\alpha})/2 \leq \beta \leq (1+\sqrt{\alpha})/2$. Together, the two conditions result in $0 < \beta \leq (1+\sqrt{\alpha})/2$. In fact, the latter gives a complete characterization of the parameter values for which *(30)* is positive for all $x \geq 0$ except possibly for one x value, given that $0 < \alpha, \beta < 1$. Finally, if $\beta = 0$ the λ_0 does not depend on n_a, so (19) is trivially satisfied for every $0 < q < 1$. □

Appendix 4. Proof of Proposition 6.

A fraction π (a fraction $1 - \pi$) of the workers has $b = b_1$ ($b = b_2$) and a fraction q (a fraction $1 - q$) of the possible firms has $p = p_1$ ($p = p_2$). The only possible wage offers are ϕ_1 and ϕ_2. The set of possible wage offer distributions F(w) can be summarized by $\Pr_F(w = \phi_1) = \gamma = 1 - \Pr_F(w = \phi_2)$ with $0 \leq \gamma \leq 1$. The parameters ϕ_1, ϕ_2 and γ remain to be determined. From equation (1) it follows that $\phi_1 = b_1$ and that

$$\phi_2 = \frac{\delta b_2 + \lambda_0 \gamma b_1}{\delta + \lambda_0 \gamma}.$$

Now let us examine the steady-state labor forces of firms paying ϕ_1 and of firms paying ϕ_2. Analogous to the derivation of equation (2), we find that the fraction γ_G of employed workers earning ϕ_1 equals

$$\gamma_G = \gamma \cdot \frac{\delta + \gamma\lambda_0 + \pi\lambda_0(1-\gamma)}{\delta + \gamma\lambda_0 - \pi\delta(1-\gamma)}$$

which is one minus the fraction earning ϕ_2. Here we use the fact that a worker is indifferent between earning ϕ_2 at a type-p, firm and earning ϕ_2 at a type-p_1 firm. Note that $\gamma_G > \gamma$, indicating that type-p_1 firms will be larger than type-p, firms.

A type-p, firm, if active, always offers ϕ_2 (recall that $p_2 < b$,). This means that the equilibrium value of γ must satisfy $\gamma \leq q$. Now consider a type-p_1 firm. If it offers b_1 then the profit flow equals $(p_1 - b_1)\gamma_G(m - u)/\gamma$, whereas if it offers ϕ_2 then this flow equals $(p_1 - \phi_2)(1 - \gamma_G)(m - u)/(1 - \gamma)$. A solution for $\gamma \in (0, q)$ means that type-p, are indifferent between offering either of these wages. The profit flows then must be equal. After some tedious calculations this can be shown to imply that

$$\pi(\delta + \lambda_0)(p_1 - b_2) = (6 + \pi\lambda_0)(b_1 - b_2) \tag{31}$$

Perhaps surprisingly, this does not depend on γ at all. Thus, in the pathological case in which this equality is satisfied, every value of γ in [0, q] admits an equilibrium. Consequently, we then have an infinite number of equilibria (this peculiar result is however not robust with respect to the inclusion of a discount rate in the individuals' optimization problem; see Mortensen and Pissarides, 1998). If the left-hand side of (31) is smaller than the right-hand side, then offering ϕ_2 is always more profitable than offering b,. Consequently, the equilibrium value of γ is zero, and ϕ_2 is driven down to b_2. Note that this is true regardless of the value of q, that is, regardless of whether there are any active type-p, firms Consequently, if the left-hand side of (31) is smaller than the right-hand side, then the unique equilibrium is that all wages equal b,. All firms are then active, but all individuals with $b = b_1$ are permanently unemployed.

Now suppose that the left-hand side of (31) exceeds the right-hand side. We start assuming that firms of both types are active in equilibrium. This must be verified, by checking whether in equilibrium $\phi_2 < p_2$. Now, b_1 is always the most profitable wage offer for type-p, firms. Consequently, the equilibrium γ equals q, and ϕ_2 equals $(\delta b_2 + \lambda_0 q b_1)/(\delta + \lambda_0 q)$. In sum, all type-p, firms offer b_1 and all type-p, firms offer this value of ϕ_2. There holds that $\phi_2 < p_2$ iff the first inequality of (25) holds.

Now again suppose that the left-hand side of (31) exceeds the right-hand side, but now assume that only type-p, firms are active in equilibrium. This must be verified, by checking whether in equilibrium $\phi_2 \geq p_2$. Again, b_1 is always the most profitable wage offer for type-p_1 firms. Consequently, the equilibrium γ equals 1, and ϕ_2 equals $(\delta b_2 + \lambda_0 b_1)/(\delta + \lambda_0)$. There holds that $\phi_2 \geq p_2$ iff the second inequality of (25) holds. This proves the proposition. • I